

*Coupon Collector's Problem for  
Fault Analysis against AES  
High Tolerance for Noisy Fault Injections*

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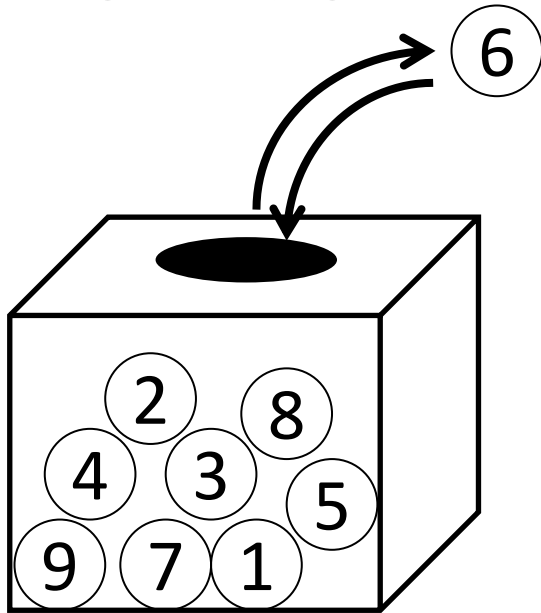
# Research Summary

- Improve a side-channel analysis (SQUARE fault analysis) against AES.
- A key is recovered even if undesired fault injection (noise) occur with some probability.
- The attack is evaluated with coupon collector's problem.

Ref.	#desired fault	#noise	complexity
[PY06]	256	<b>0</b>	$2^{37}$
[K10]	44	<b>0</b>	$2^{34}$
<b>Ours</b>	256	<b>1610</b>	$2^{45}$
	128	<b>49</b>	$2^{41}$

# Coupon Collector's Problem (CCP)

- Definition



For each coupon drawing event, 1 random coupon is obtained.

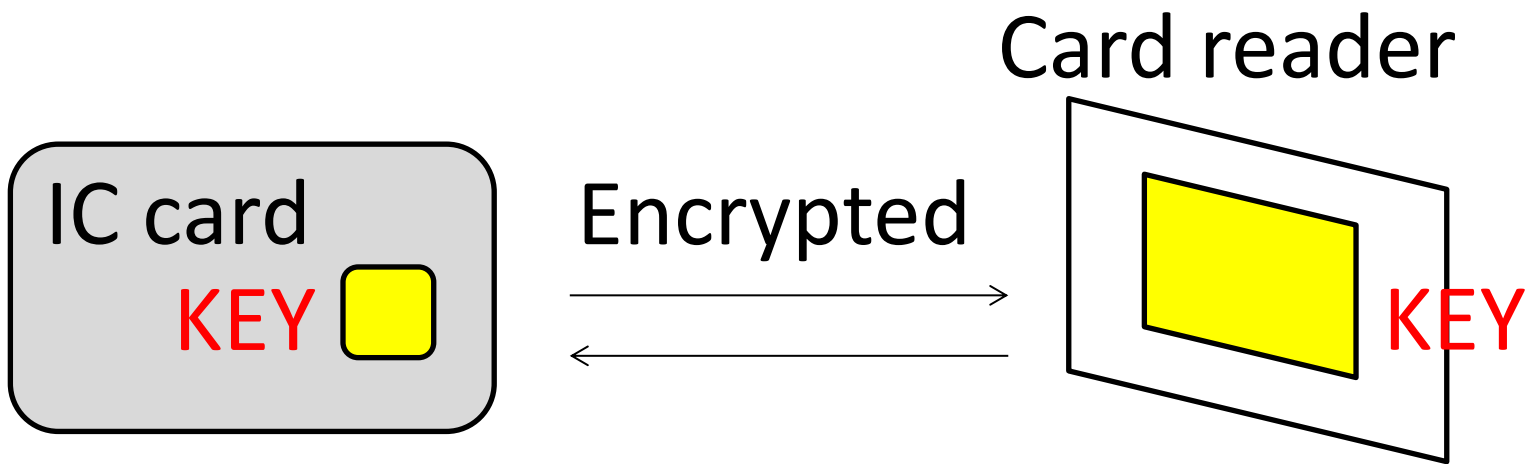
How many events are expected to complete all coupons?

$$n \ln(n)$$

- CCP can be applied to the fault attack.

# Symmetric-key Encryption in Practice

- Symmetric-key encryption is widely used to protect the communication.



- AES is the most popular algorithm.
- Its implementation needs to be protected.

# Fault Attack

- A kind of side-channel analysis.
- Give some external factor during the encryption computation to make some error.
  - Laser irradiation: give extra energy to flip internal state bits.
  - Clock glitch: force to start the next computation before the previous computation is finished.

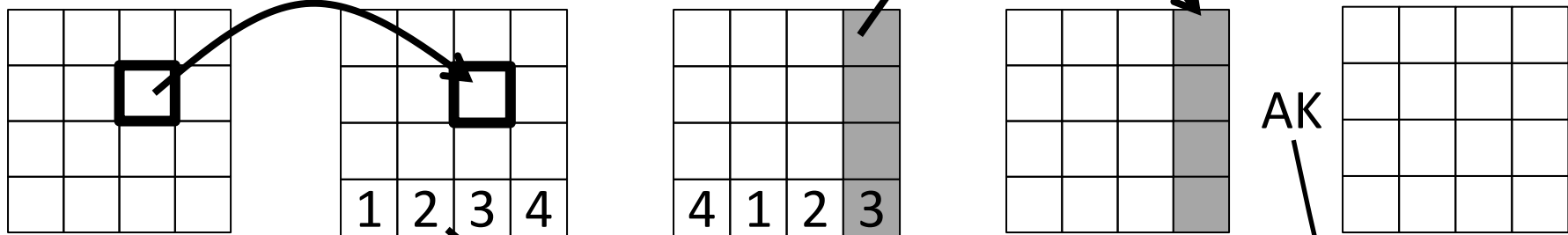


# AES

- 128-bit block-cipher
- Standardized and used all over the world
- Mix 16-byte data with 10 rounds
- Computations in each round is as follows.

MC: Column-wise linear operation

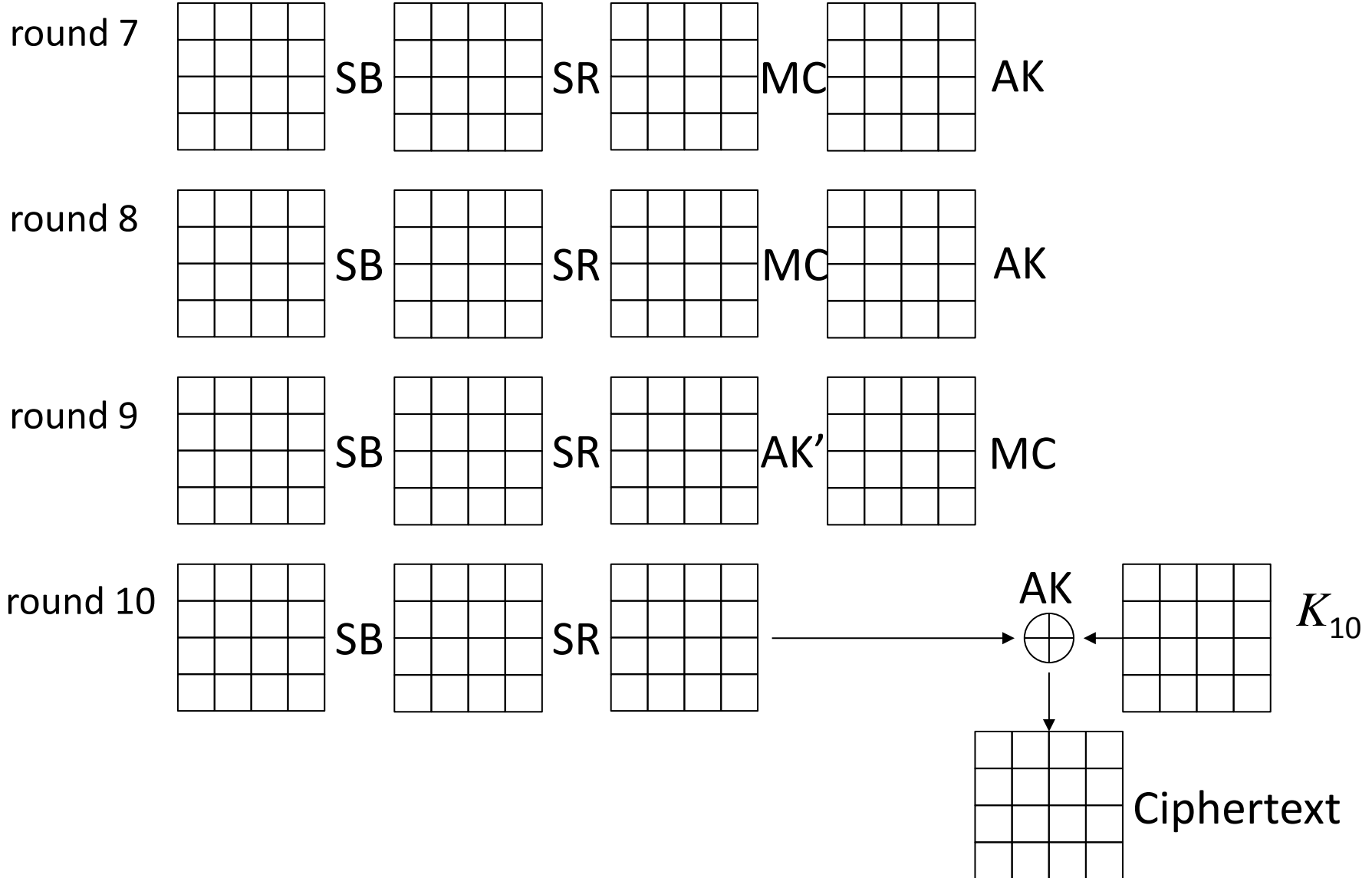
SB: byte-wise permutation



SR: Row-wise position exchange

XOR with round key <sub>6</sub>

# NTT The Last 4 Rounds of AES

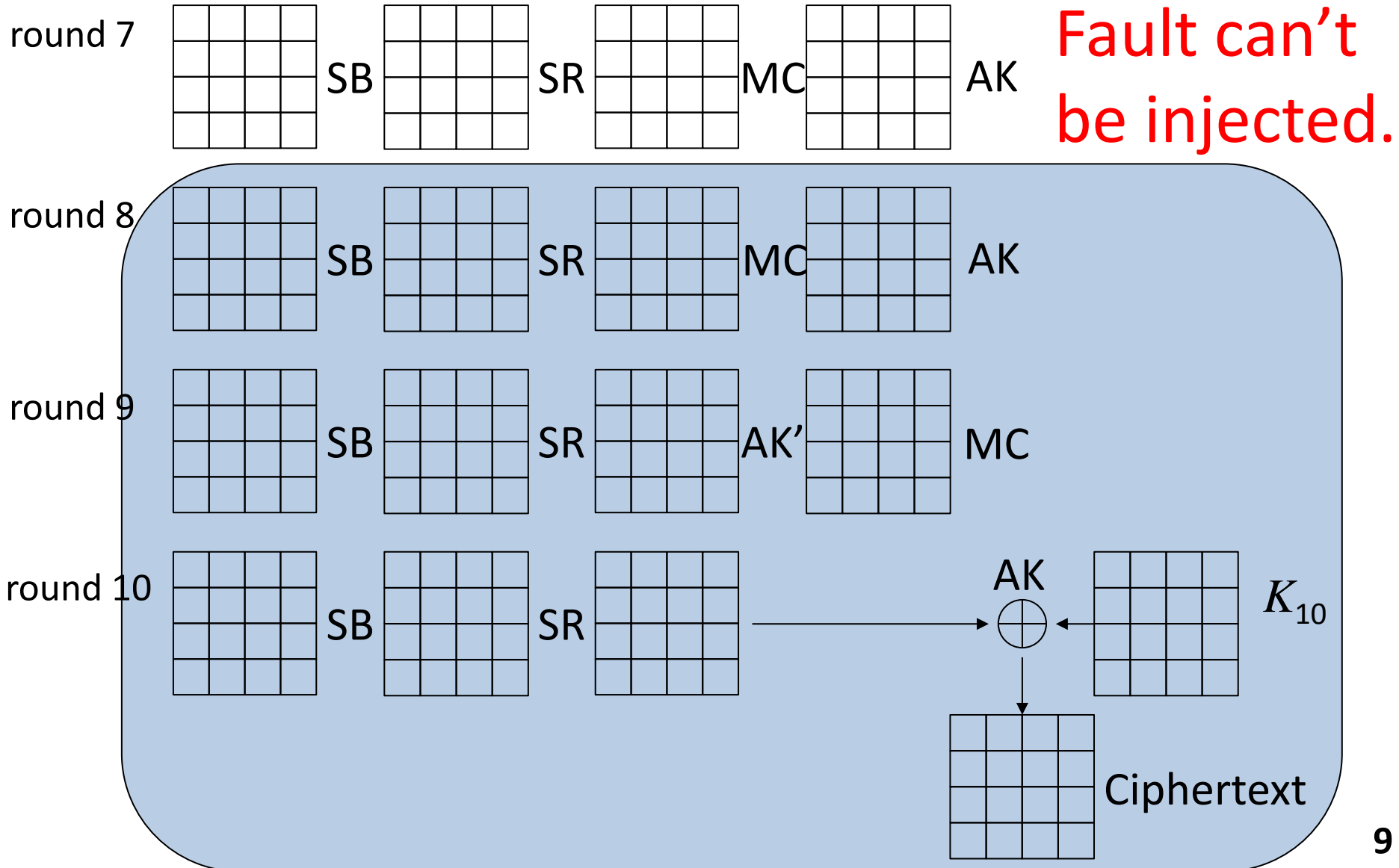


# DFA and Its Countermeasure

- Differential fault analysis (DFA) is famous as a very powerful attack.
- If a fault is injected during the last 3 rounds of AES, the key is recovered easily.
- Countermeasures against fault analysis are expensive (overhead is 200%).
- It's natural to minimize the location to be protected: only the last 3 rounds.



# NTT The Last 4 Rounds of AES



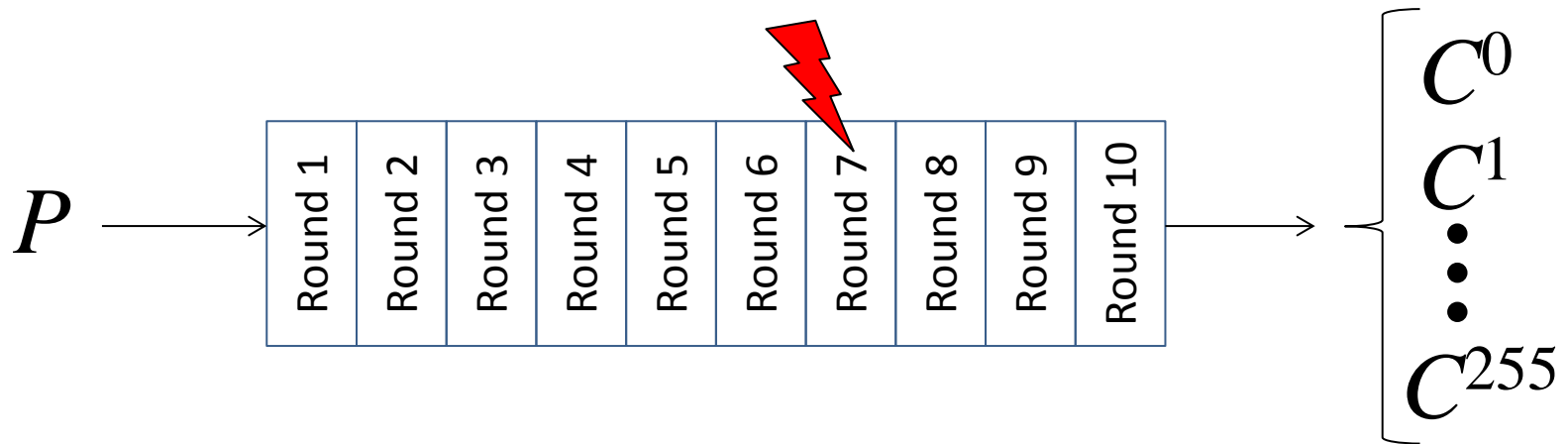
# Research Motivation

- Phan and Yin showed that the key is recovered even with the fault in round 7.
- Do we need to protect round 7 as well?
- Unfortunately, their attack assumption (fault model) is very strong.

**In this research, we relax the assumption!!**

# SQUARE DFA [PhanYin06]

- While the same plaintext is encrypted 256 times, a byte in round 7 is forced to take all 256 values by using the fault.



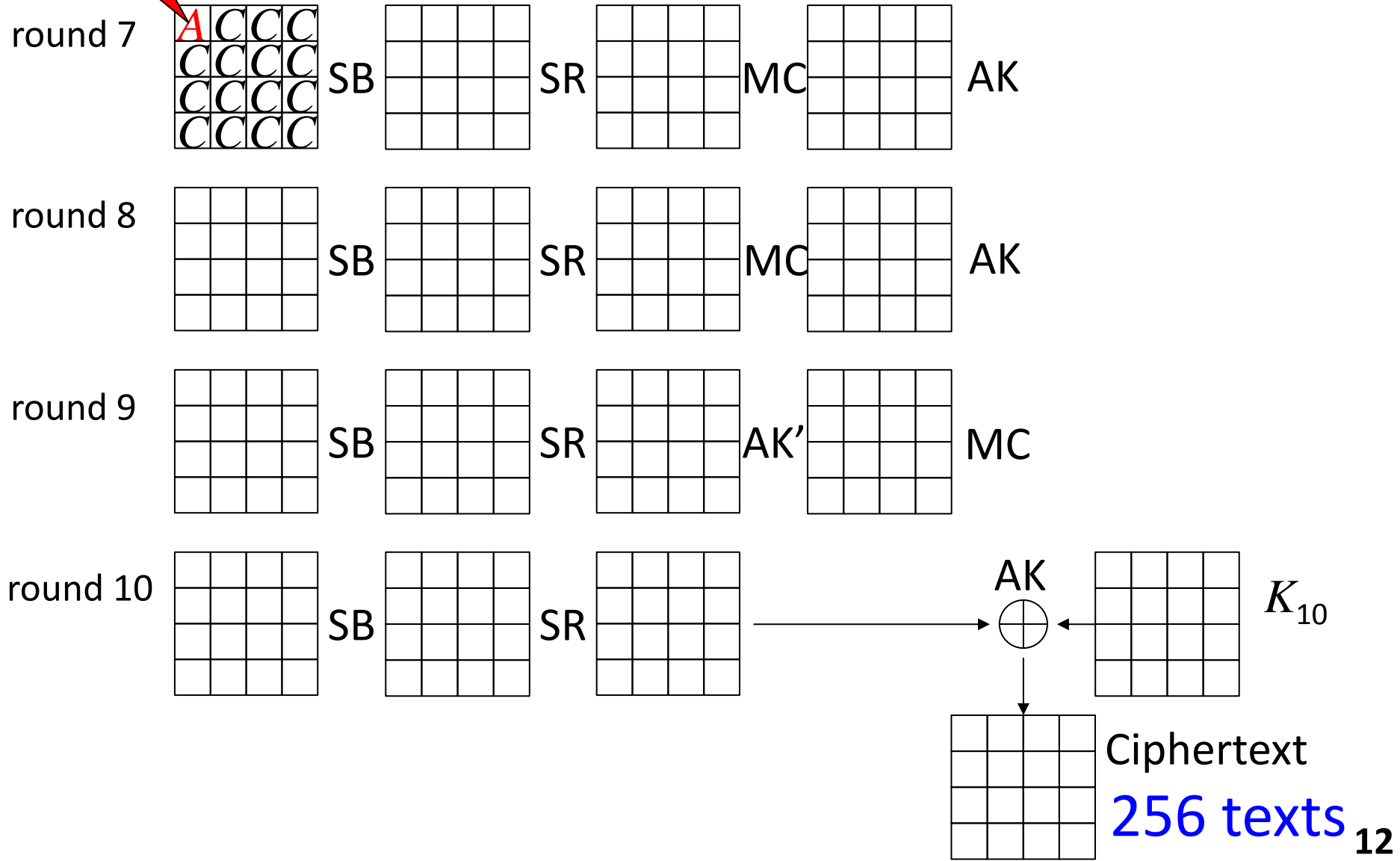
Fault model:

- *The attacker can flip any bit*
- *Undesired fault (noise) never occurs*



# SQUARE DFA [PhanYin06]

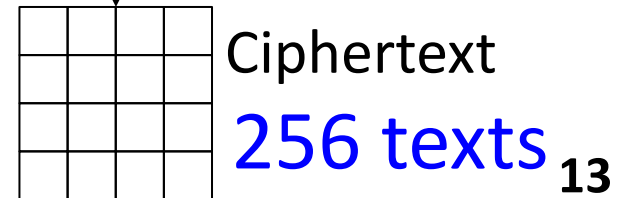
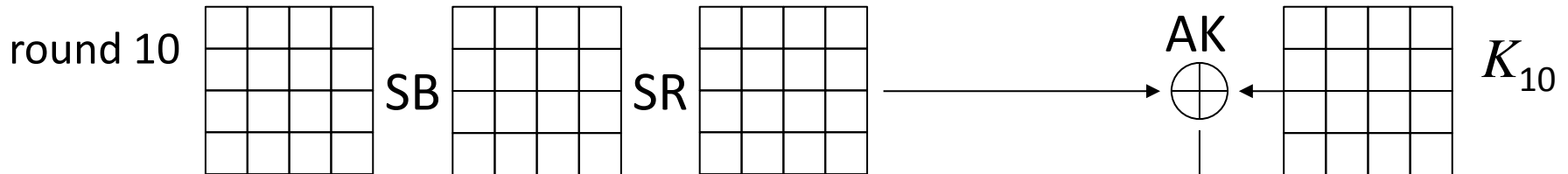
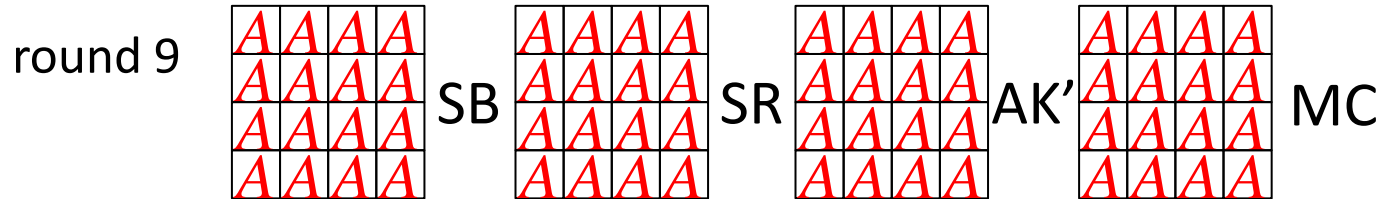
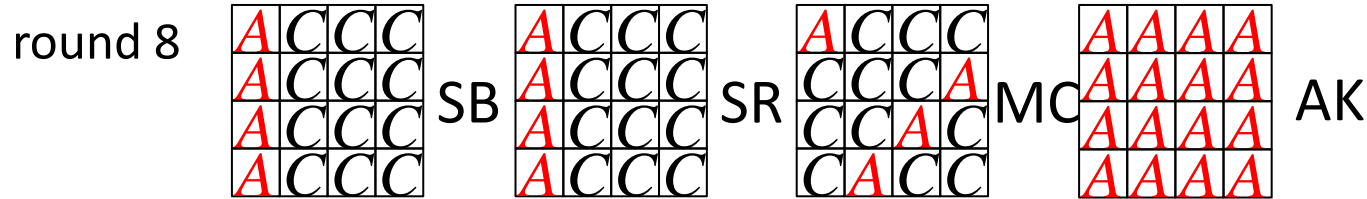
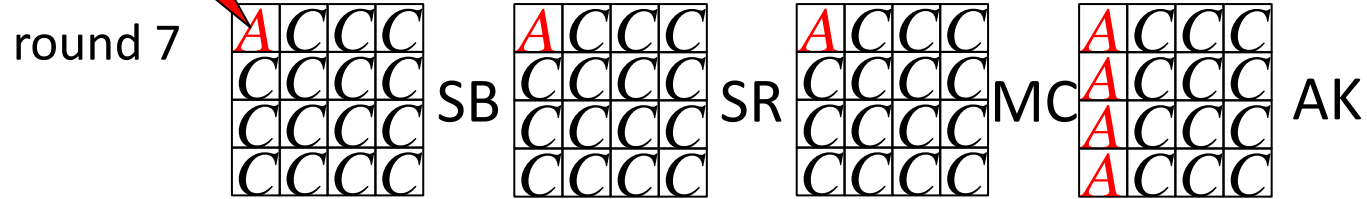
 Collect all values by fault injections.



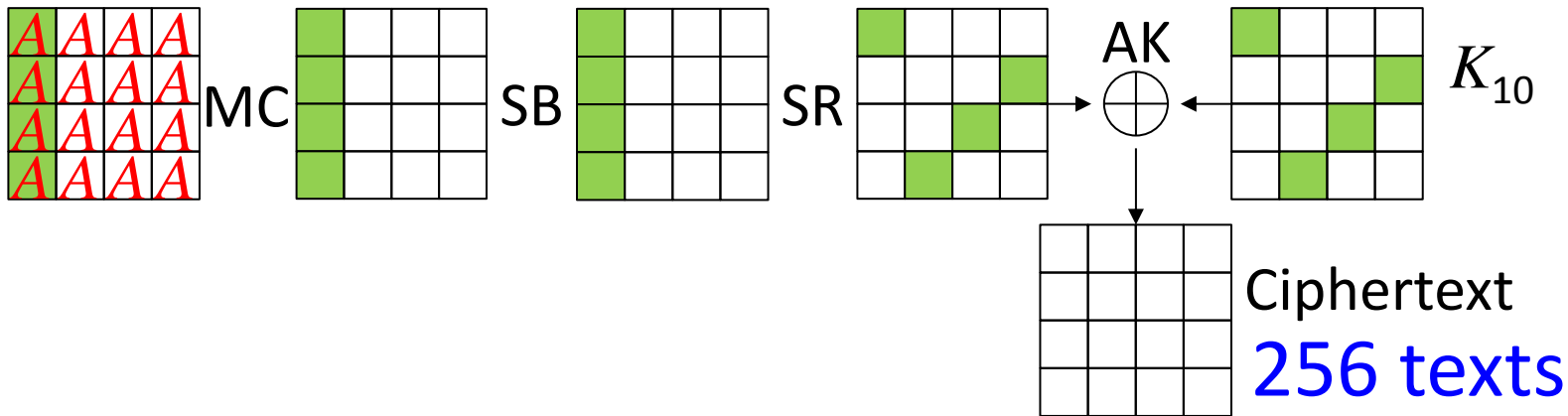


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# SQUARE DFA [PhanYin06]

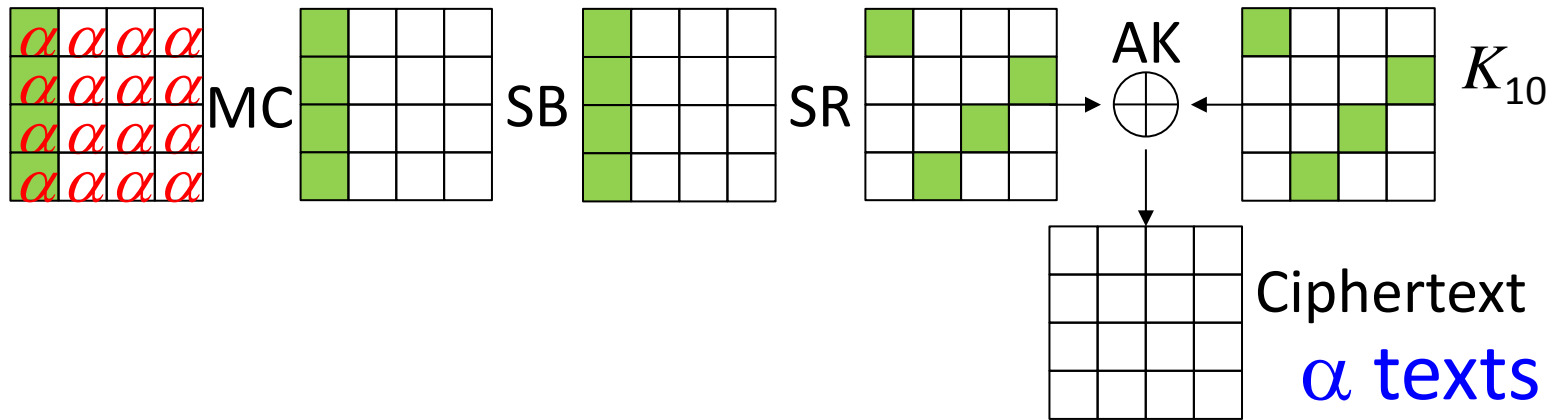


The key  $K_{10}$  is guessed column by column.

If the guess is correct, each byte takes all 256 distinct values after the 1 round decryption.

Probability: 
$$\left( \prod_{i=0}^{255} \frac{(256 - i)}{256} \right)^4$$

# Improved SQUARE DFA [Kim11]



256 values are not necessary.  $\alpha$  values are enough.

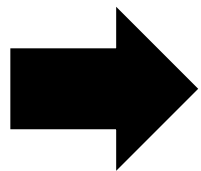
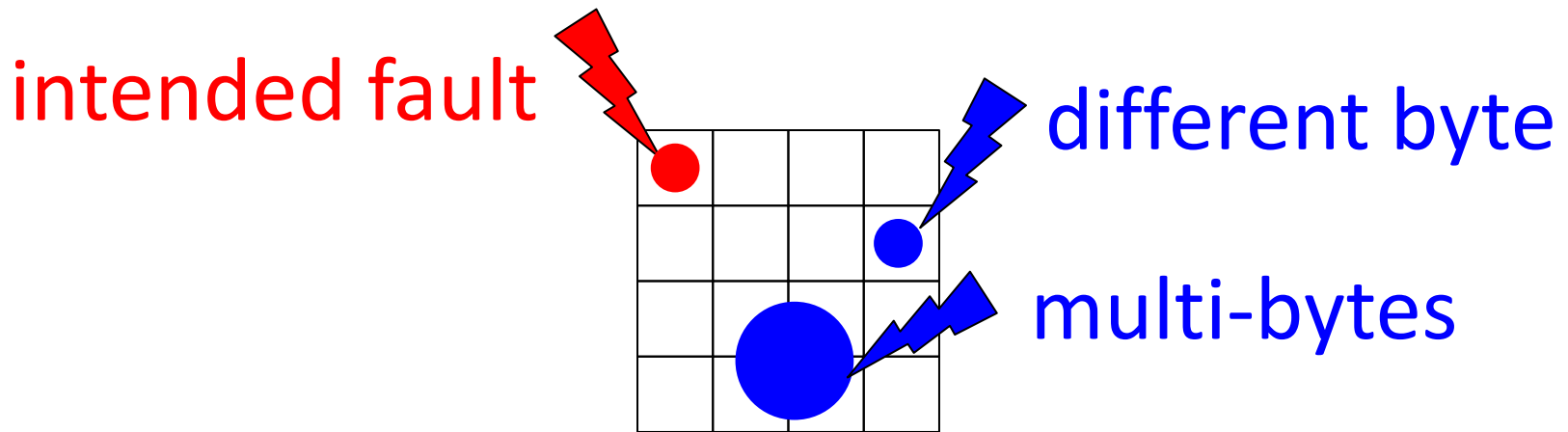
For the correct guess, each byte takes  $\alpha$  values.

Probability: 
$$\left( \prod_{i=0}^{\alpha-1} \frac{(256 - i)}{256} \right)^4$$

The probability is smaller than  $2^{-32}$  for  $\alpha = 44$ .

# Noisy Fault Model

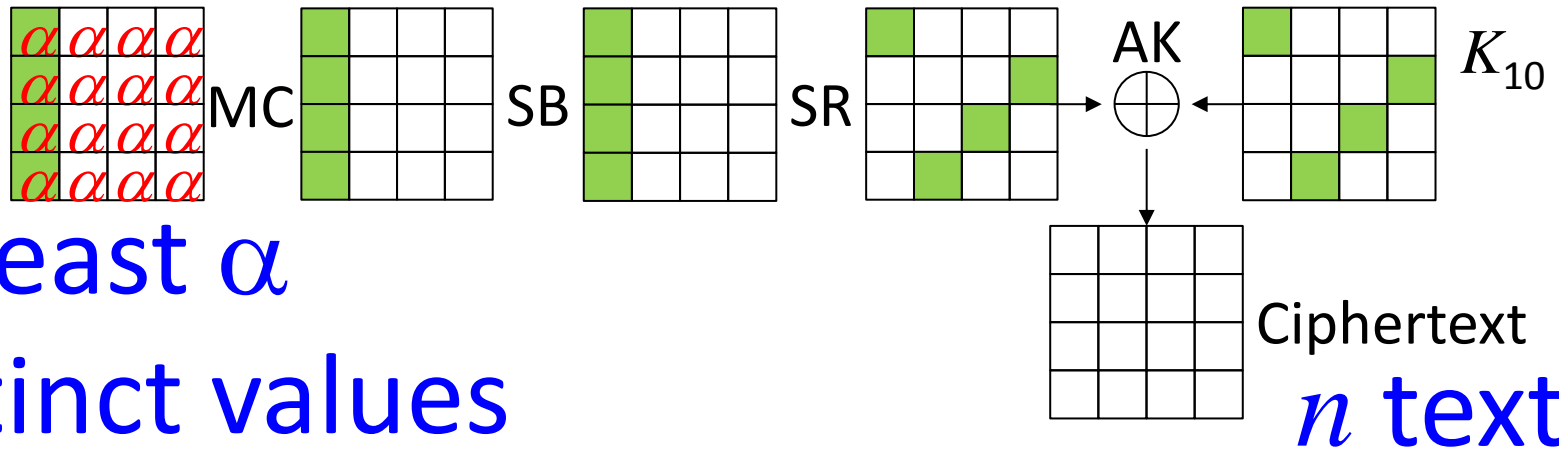
- Previous SQUARE DFAs assume that unintended fault never occurs.
- But, in practice, noise is obtained.



We can still recover the key !!



# Our Attacks



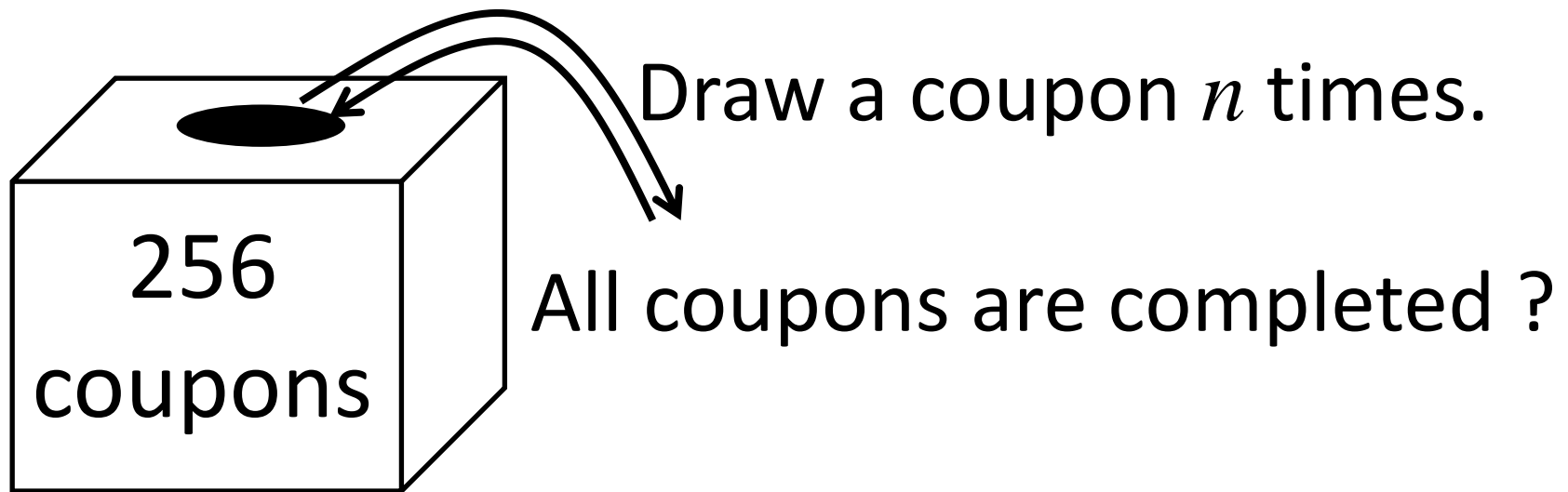
- $\alpha$ : the number of distinct fault values
- $n$ : the total number of texts to be analyzed

For the correct guess at least  $\alpha$  distinct values appear, otherwise, the guess is wrong.

**What's the probability?**

# Probability Estimation with CCP

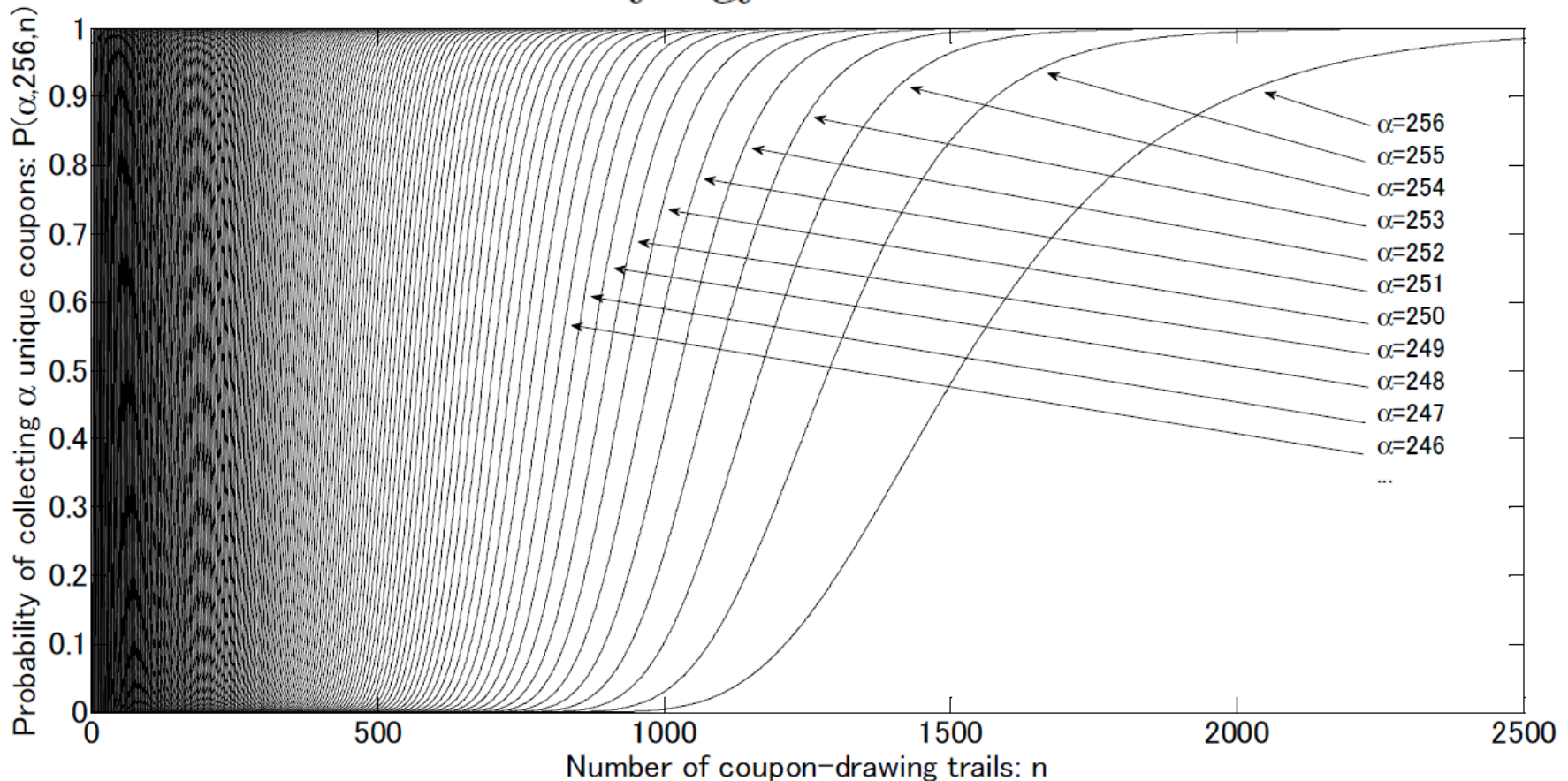
- Suppose that  $\alpha = 256$ . Each guess is a right key candidate if all 256 values are completed after  $n$  trials.



- equivalent to the CCP.  $\Pr = 2^{-1}$  even if  $n = 1553$ .
- For  $\alpha < 256$ , it becomes a variant of the CCP.

# Probability Estimation with CCP

$$\binom{\beta}{\alpha} \binom{\alpha}{1} \sum_{i=\alpha}^n \frac{Q(\alpha - 1, i - 1)}{\beta^i}$$



# Example Parameters

Value of  $n$

	$\alpha = 64$	$\alpha = 128$	$\alpha = 256$
$P(\alpha, 256, n)^4 = 2^{-1}$	77	186	1866
$P(\alpha, 256, n)^4 = 2^{-4}$	73	177	1553
$P(\alpha, 256, n)^4 = 2^{-32}$	66	156	933

# Conclusion

- We generalized the SQUARE DFA so that the noisy fault injection can be accepted.
- We did the probability estimation with the coupon collector's problem.
- Possible future direction
  - Detect a suitable fault injection method.
  - Evaluate other ciphers.

***Thank you for your attention !!***